Division by Zero

By Ted Chiang

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Stories of Your Life and Others Ted Chiang

1

Dividing a number by zero doesn't produce an infinitely large number as an answer. The reason is that division is defined as the inverse of multiplication; if you divide by zero, and then multiply by zero, you should regain the number you started with. However, multiplying infinity by zero produces only zero, not any other number. There is nothing which can be multiplied by zero to produce a nonzero result; therefore, the result of a division by zero is literally "undefined."

1a

Renee was looking out the window when Mrs. Rivas approached.

"Leaving after only a week? Hardly a real stay at all. Lord knows I won't be leaving for a long time."

Renee forced a polite smile. "I'm sure it won't be long for you." Mrs. Rivas was the manipulator in the ward; everyone knew that her attempts were merely gestures, but the aides wearily paid attention to her lest she succeed accidentally.

"Ha. They wish I'd leave. You know what kind of liability they face if you die while you're on status?"

"Yes, I know."

"That's all they're worried about, you can tell. Always their liability -- "

Renee tuned out and returned her attention to the window, watching a contrail extrude itself across the sky.

"Mrs. Norwood?" a nurse called. "Your husband's here."

Renee gave Mrs. Rivas another polite smile and left.

1b

Carl signed his name yet another time, and finally the nurses took away the forms for processing.

He remembered when he had brought Renee in to be admitted, and thought of all the stock questions at the first interview. He had answered them all stoically.

"Yes, she's a professor of mathematics. You can find her in Who's Who."

"No, I'm in biology."

And:

"I had left behind a box of slides that I needed."

"No, she couldn't have known."

And, just as expected:

"Yes, I have. It was about twenty years ago, when I was a grad student."

"No, I tried jumping."

"No, Renee and I didn't know each other then."

And on and on.

Now they were convinced that he was competent and supportive, and were ready to release Renee into an outpatient treatment program.

Looking back, Carl was surprised in an abstracted way. Except for one moment, there hadn't been any sense of deja vu at any time during the entire ordeal. All the time he was dealing with the hospital, the doctors, the nurses: the only accompanying sensation was one of numbness, of sheer tedious rote.

2

There is a well-known "proof" that demonstrates that one equals two. It begins with some definitions: "Let a = 1; let b = 1." It ends with the conclusion "a = 2a," that is, one equals two. Hidden inconspicuously in the middle is a division by zero, and at that point the proof has stepped off the brink, making all rules null and void. Permitting division by zero allows one to prove not only that one and two are equal, but that any two numbers at all -- real or imaginary, rational or irrational -- are equal.

2a

As soon as she and Carl got home, Renee went to the desk in her study and began turning all the papers facedown, blindly sweeping them together into a pile; she winced whenever a corner of a page faced up during her shuffling. She considered burning the pages, but that would be merely symbolic now. She'd accomplish as much by simply never glancing at them.

The doctors would probably describe it as obsessive behavior. Renee frowned, reminded of the indignity of being a patient under such fools. She remembered being on suicide status, in the locked ward, under the supposedly round-the-clock observation of the aides. And the interviews with the doctors, who were so condescending, so obvious. She was no manipulator like Mrs. Rivas, but it really was easy. Simply say "I realize I'm not well yet, but I do feel better," and you'd be considered almost ready for release.

Carl watched Renee from the doorway for a moment, before he passed down the hallway. He remembered the day, fully two decades past, when he himself had been released. His parents had picked him up, and on the trip back his mother had made some inane comment about how glad everyone would be to see him, and he was just barely able to restrain himself from shaking her arm off his shoulders.

He had done for Renee what he would have appreciated during his period under observation. He had come to visit every day, even though she refused to see him at first, so that he wouldn't be absent when she did want to see him. Sometimes they talked, and sometimes they simply walked around the grounds. He could find nothing wrong in what he did, and he knew that she appreciated it.

Yet, despite all his efforts, he felt no more than a sense of duty towards her.

3

In the *Principia Mathematica*, Bertrand Russell and Alfred Whitehead attempted to give a rigorous foundation to mathematics using formal logic as their basis. They began with what they considered to be axioms, and used those to derive theorems of increasing complexity. By page 362, they had established enough to prove "1 + 1 = 2."

3a

As a child of seven, while investigating the house of a relative, Renee had been spellbound at discovering the perfect squares in the smooth marble tiles of the floor. A single one, two rows of two, three rows of three, four rows of four: the tiles fit together in a *square*. Of course. No matter which side you looked at it from, it came out the same. And more than that, each square was bigger than the last by an *odd number of tiles*. It was an epiphany. The conclusion was necessary: it had a rightness to it, confirmed by the smooth, cool feel of the tiles. And the way the tiles were fitted together, with such incredibly fine lines where they met; she had shivered at the precision.

Later on there came other realizations, other achievements. The astonishing doctoral dissertation at twenty-three, the series of acclaimed papers; people compared her to Von Neumann, universities wooed her. She had never paid any of it much attention. What she did pay attention to was that same sense of rightness, possessed by every theorem she learned, as insistent as the tiles' physicality, and as exact as their fit.

3b

Carl felt that the person he was today was born after his attempt, when he met Laura. After being released from the hospital, he was in no mood to see anyone, but a friend of his had managed to introduce him to Laura. He had pushed her away initially, but she had known better. She had loved him while he was hurting, and let him go once he was healed. Through knowing her Carl had learned about empathy, and he was remade.

Laura had moved on after getting her own master's degree, while he stayed at the university for his doctorate in biology. He suffered various crises and heartbreaks later on in life, but never again despair.

Carl marveled when he thought about what kind of person she was. He hadn't spoken to her since grad school; what had her life been like over the years? He wondered whom else she had loved. Early on he had recognized what kind of love it was, and what kind it wasn't, and he valued it immensely.

2b

In the early nineteenth century, mathematicians began exploring geometries that differed from Euclidean geometry; these alternate geometries produced results that seemed utterly absurd, but they didn't produce logical contradictions. It was later shown that these non-Euclidean geometries were consistent relative to Euclidean geometry: they were logically consistent, as long as one assumed that Euclidean geometry was consistent.

The proof of Euclidean geometry's consistency eluded mathematicians. By the end of the nineteenth century, the best that was achieved was a proof that Euclidean geometry was consistent as long as arithmetic was consistent.

4a

At the time, when it all began, Renee had thought it little more than an annoyance. She had walked down the hall and knocked on the open door of Peter Fabrisi's office. "Pete, got a minute?"

Fabrisi pushed his chair back from his desk. "Sure, Renee, what's up?"

Renee came in, knowing what his reaction would be. She had never asked anyone in the department for advice on a problem before; it had always been the reverse. No matter. "I was wondering if you could do me a favor. You remember what I was telling you about a couple weeks back, about the formalism I was developing?"

He nodded. "The one you were rewriting axiom systems with."

"Right. Well, a few days ago I started coming up with really ridiculous conclusions, and now my formalism is contradicting itself. Could you take a look at it?"

Fabrisi's expression was as expected. "You want -- sure, I'd be glad to."

"Great. The examples on the first few pages are where the problem is; the rest is just for your reference." She handed Fabrisi a thin sheaf of papers. "I thought if I talked you through it, you'd just see the same things I do."

"You're probably right." Fabrisi looked at the first couple pages. "I don't know how long this'll take."

"No hurry. When you get a chance, just see whether any of my assumptions seem a little dubious, anything like that. I'll still be going at it, so I'll tell you if I come up with anything. Okay?"

Fabrisi smiled. "You're just going to come in this afternoon and tell me you've found the problem."

"I doubt it: this calls for a fresh eye."

He spread his hands. "I'll give it a shot."

"Thanks." It was unlikely that Fabrisi would fully grasp her formalism, but all she needed was someone who could check its more mechanical aspects.

4b

Carl had met Renee at a party given by a colleague of his. He had been taken with her face. Hers was a remarkably plain face, and it appeared quite somber most of the time, but during the party he saw her smile twice and frown once; at those moments, her entire countenance assumed the expression as if it had never known another. Carl had been caught by surprise: he could recognize a face that smiled regularly,

4

or a face that frowned regularly, even if it were unlined. He was curious as to how her face had developed such a close familiarity with so many expressions, and yet normally revealed nothing.

It took a long time for him to understand Renee, to read her expressions. But it had definitely been worthwhile.

Now Carl sat in his easy chair in his study, a copy of the latest issue of *Marine Biology* in his lap, and listened to the sound of Renee crumpling paper in her study across the hall. She'd been working all evening, with audibly increasing frustration, though she'd been wearing her customary poker face when last he'd looked in.

He put the journal aside, got up from the chair, and walked over to the entrance of her study. She had a volume opened on her desk; the pages were filled with the usual hieroglyphic equations, interspersed with commentary in Russian.

She scanned some of the material, dismissed it with a barely perceptible frown, and slammed the volume closed. Carl heard her mutter the word "useless," and she returned the tome to the bookcase.

"You're gonna give yourself high blood pressure if you keep up like this," Carl jested.

"Don't patronize me."

Carl was startled. "I wasn't."

Renee turned to look at him and glared. "I know when I'm capable of working productively and when I'm not."

Chilled. "Then I won't bother you." He retreated.

"Thank you." She returned her attention to the bookshelves. Carl left, trying to decipher that glare.

5

At the Second International Congress of Mathematics in 1900, David Hilbert listed what he considered to be the twenty-three most important unsolved problems of mathematics. The second item on his list was a request for a proof of the consistency of arithmetic. Such a proof would ensure the consistency of a great deal of higher mathematics. What this proof had to guarantee was, in essence, that one could never prove one equals two. Few mathematicians regarded this as a matter of much import.

5a

Renee had known what Fabrisi would say before he opened his mouth.

"That was the damnedest thing I've ever seen. You know that toy for toddlers where you fit blocks with different cross sections into the differently shaped slots? Reading your formal system is like watching someone take one block and sliding it into every single hole on the board, and making it a perfect fit every time."

"So you can't find the error?"

He shook his head. "Not me. I've slipped into the same rut as you. I can only think about it one way."

Renee was no longer in a rut: she had come up with a totally different approach to the question, but it only confirmed the original contradiction. "Well, thanks for trying."

"You going to have someone else take a look at it?"

"Yes, I think I'll send it to Callahan over at Berkeley. We've been corresponding since the conference last spring."

Fabrisi nodded. "I was really impressed by his last paper. Let me know if he can find it: I'm curious."

Renee would have used a stronger word than "curious" for herself.

5b

Was Renee just frustrated with her work? Carl knew that she had never considered mathematics really difficult, just intellectually challenging. Could it be that for the first time she was running into problems that she could make no headway against? Or did mathematics work that way at all? Carl himself was strictly an experimentalist; he really didn't know how Renee made new math. It sounded silly, but perhaps she was running out of ideas?

Renee was too old to be suffering from the disillusionment of a child prodigy becoming an average adult. On the other hand, many mathematicians did their best work before the age of thirty, and she might be growing anxious over whether that statistic was catching up to her, albeit several years behind schedule.

It seemed unlikely. He gave a few other possibilities cursory consideration. Could she be growing cynical about academia? Dismayed that her research had become overspecialized? Or simply weary of her work?

Carl didn't believe that such anxieties were the cause of Renee's behavior; he could imagine the impressions that he would pick up if that were the case, and they didn't mesh with what he was receiving. Whatever was bothering Renee, it was something he couldn't fathom, and that disturbed him.

6

In 1931, Kurt Godel demonstrated two theorems. The first one shows, in effect, that mathematics contains statements that may be true, but are inherently unprovable. Even a formal system as simple as arithmetic permits statements that are precise, meaningful, and seem certainly true, and yet cannot be proven true by formal means.

His second theorem shows that a claim of the consistency of arithmetic is just such a statement; it cannot be proven true by any means using the axioms of arithmetic. That is, arithmetic as a formal system cannot guarantee that it will not produce results such as

"1 = 2"; such contradictions may never have been encountered, but it is impossible to prove that they never will be.

6a

Once again, he had come into her study. Renee looked up from her desk at Carl; he began resolutely, "Renee, it's obvious that--"

She cut him off. "You want to know what's bothering me? Okay, I'll tell you." Renee got out a blank sheet of paper and sat down at her desk. "Hang on; this'll take a minute." Carl opened his mouth again, but Renee waved him silent. She took a deep breath and began writing.

She drew a line down the center of the page, dividing it into two columns. At the head of one column she wrote the numeral "1" and for the other she wrote "2". Below them she rapidly scrawled out some

symbols, and in the lines below those she expanded them into strings of other symbols. She gritted her teeth as she wrote: forming the characters felt like dragging her fingernails across a chalkboard.

About two thirds of the way down the page, Renee began reducing the long strings of symbols into successively shorter strings. *And now for the masterstroke*, she thought. She realized she was pressing hard on the paper; she consciously relaxed her grip on the pencil. On the next line that she put down, the strings became identical. She wrote an emphatic "=" across the center line at the bottom of the page.

She handed the sheet to Carl. He looked at her, indicating incomprehension. "Look at the top." He did so. "Now look at the bottom."

He frowned. "I don't understand."

"I've discovered a formalism that lets you equate any number with any other number. That page there proves that one and two are equal. Pick any two numbers you like; I can prove those equal as well."

Carl seemed to be trying to remember something. "It's a division by zero, right?"

"No. There are no illegal operations, no poorly defined terms, no independent axioms that are implicitly assumed, nothing. The proof employs absolutely nothing that's forbidden."

Carl shook his head. "Wait a minute. Obviously one and two aren't the same."

"But formally they are: the proof's in your hand. Everything I've used is within what's accepted as absolutely indisputable."

"But you've got a contradiction here."

"That's right. Arithmetic as a formal system is inconsistent."

6b

"You can't find your mistake, is that what you mean?"

"*No*, you're not listening. You think I'm just frustrated because of something like that? There is no mistake in the proof."

"You're saying there's something wrong within what's accepted?"

"Exactly."

"Are you--" He stopped, but too late. She glared at him. Of course she was sure. He thought about what she was implying.

"Do you see?" asked Renee. "I've just disproved most of mathematics: it's all meaningless now."

She was getting agitated, almost distraught; Carl chose his words carefully. "How can you say that? Math still works. The scientific and economic worlds aren't suddenly going to collapse from this realization."

"That's because the mathematics they're using is just a gimmick. It's a mnemonic trick, like counting on your knuckles to figure out which months have thirty-one days."

"That's not the same."

"Why isn't it? Now mathematics has absolutely nothing to do with reality. Never mind concepts like

imaginaries or infinitesimals. Now goddamn integer addition has nothing to do with counting on your fingers. One and one will always get you two on your fingers, but on paper I can give you an infinite number of answers, and they're all equally valid, which means they're all equally invalid. I can write the most elegant theorem you've ever seen, and it won't mean any more than a nonsense equation." She gave a bitter laugh. "The positivists used to say all mathematics is a tautology. They had it all wrong: it's a contradiction."

Carl tried a different approach. "Hold on. You just mentioned imaginary numbers. Why is this any worse than what went on with those? Mathematicians once believed they were meaningless, but now they're accepted as basic. This is the same situation."

"It's *not* the same. The solution there was to simply expand the context, and that won't do any good here. Imaginary numbers added something new to mathematics, but my formalism is redefining what's already there."

"But if you change the context, put it in a different light--"

She rolled her eyes. "No! This follows from the axioms as surely as addition does; there's no way around it. You can take my word for it."

7

In 1936, Gerhard Gentzen provided a proof of the consistency of arithmetic, but to do it he needed to use a controversial technique known as transfinite induction. This technique is not among the usual methods of proof, and it hardly seemed appropriate for guaranteeing the consistency of arithmetic. What Gentzen had done was prove the obvious by assuming the doubtful.

7a

Callahan had called from Berkeley, but could offer no rescue. He said he would continue to examine her work, but it seemed that she had hit upon something fundamental and disturbing. He wanted to know about her plans for publication of her formalism, because if it did contain an error that neither of them could find, others in the mathematics community would surely be able to.

Renee had barely been able to hear him speaking, and mumbled that she would get back to him. Lately she had been having difficulty talking to people, especially since the argument with Carl; the other members of the department had taken to avoiding her. Her concentration was gone, and last night she had had a nightmare about discovering a formalism that let her translate arbitrary concepts into mathematical expressions: then she had proven that life and death were equivalent.

That was something that frightened her: the possibility that she was losing her mind. She was certainly losing her clarity of thought, and that came pretty close.

What a ridiculous woman you are, she chided herself. Was Godel suicidal after he demonstrated his incompleteness theorem?

But that was beautiful, numinous, one of the most elegant theorems Renee had ever seen.

Her own proof taunted her, ridiculed her. Like a brainteaser in a puzzle book, it said gotcha, you skipped right over the mistake, see if you can find where you screwed up; only to turn around and say, gotcha again.

She imagined Callahan would be pondering the implications that her discovery held for mathematics. So much of mathematics had no practical application; it existed solely as a formal theory, studied for its

intellectual beauty. But that couldn't last; a self-contradictory theory was so pointless that most mathematicians would drop it in disgust.

What truly infuriated Renee was the way her own intuition had betrayed her. The damned theorem made sense; in its own perverted way, it *felt right*. She understood it, knew why it was true, believed it.

7b

Carl smiled when he thought of her birthday.

"I can't believe you! How could you possibly have known?" She had run down the stairs, holding a sweater in her hands.

Last summer they had been in Scotland on vacation, and in one store in Edinburgh there had been a sweater that Renee had been eyeing but didn't buy. He had ordered it, and placed it in her dresser drawer for her to find that morning.

"You're just so transparent," he had teased her. They both knew that wasn't true, but he liked to tell her that.

That was two months ago. A scant two months.

Now the situation called for a change of pace. Carl went into her study, and found Renee sitting in her chair, staring out the window. "Guess what I got for us."

She looked up. "What?"

"Reservations for the weekend. A suite at the Biltmore. We can relax and do absolutely nothing--"

"Please stop," Renee said. "I know what you're trying to do, Carl. You want us to do something pleasant and distracting to take my mind off this formalism. But it won't work. You don't know what kind of hold this has on me."

"Come on, come on." He tugged at her hands to get her off the chair, but she pulled away. Carl stood there for a moment, when suddenly she turned and locked eyes with him.

"You know I've been tempted to take barbiturates? I almost wish I were an idiot, so I wouldn't have to think about it."

He was taken aback. Uncertain of his bearings, he said, "Why won't you at least try to get away for a while? It couldn't hurt, and maybe it'll take your mind off this."

"It's not anything I can take my mind off of. You just don't understand."

"So explain it to me."

Renee exhaled and turned away to think for a moment. "It's like everything I see is shouting the contradiction at me," she said. "I'm equating numbers all the time now."

Carl was silent. Then, with sudden comprehension, he said, "Like the classical physicists facing quantum mechanics. As if a theory you've always believed has been superseded, and the new one makes no sense, but somehow all the evidence supports it."

"No, it's not like that at all." Her dismissal was almost contemptuous. "This has nothing to do with evidence; it's all a priori."

"How is that different? Isn't it just the evidence of your reasoning then?"

"Christ, are you joking? It's the difference between my measuring one and two to have the same value, and my intuiting it. I can't maintain the concept of distinct quantities in my mind anymore; they all feel the same to me."

"You don't mean that," he said. "No one could actually experience such a thing; it's like believing six impossible things before breakfast."

"How would you know what I can experience?"

"I'm trying to understand."

"Don't bother."

Carl's patience was gone. "All right then." He walked out of the room and canceled their reservations.

They scarcely spoke after that, talking only when necessary. It was three days later that Carl forgot the box of slides he needed, and drove back to the house, and found her note on the table.

Carl intuited two things in the moments following. The first came to him as he was racing through the house, wondering if she had gotten some cyanide from the chemistry department: it was the realization that, because he couldn't understand what had brought her to such an action, he couldn't feel anything for her.

The second intuition came to him as he was pounding on the bedroom door, yelling at her inside: he experienced deja vu. It was the only time the situation would feel familiar, and yet it was grotesquely reversed. He remembered being on the other side of a locked door, on the roof of a building, hearing a friend pounding on the door and yelling for him not to do it. And as he stood there outside the bedroom door, he could hear her sobbing, on the floor paralyzed with shame, exactly the same as he had been when it was him on the other side.

8

Hilbert once said, "If mathematical thinking is defective, where are we to find truth and certitude?"

8a

Would her suicide attempt brand her for the rest of her life? Renee wondered. She aligned the corners of the papers on her desk. Would people henceforth regard her, perhaps unconsciously, as flighty or unstable? She had never asked Carl if he had ever felt such anxieties, perhaps because she never held his attempt against him. It had happened many years ago, and anyone seeing him now would immediately recognize him as a whole person.

But Renee could not say the same for herself. Right now she was unable to discuss mathematics intelligibly, and she was unsure whether she ever could again. Were her colleagues to see her now, they would simply say, She's lost the knack.

Finished at her desk, Renee left her study and walked into the living room. After her formalism circulated through the academic community, it would require an overhaul of established mathematical foundations, but it would affect only a few as it had her. Most would be like Fabrisi; they would follow the proof mechanically, and be convinced by it, but no more. The only persons who would feel it nearly as keenly as she had were those who could actually grasp the contradiction, who could intuit it. Callahan was one of those; she wondered how he was handling it as the days wore on.

Renee traced a curly pattern in the dust on an end table. Before, she might have idly parameterized the curve, examined some of its characteristics. Now there seemed no point. All of her visualizations simply collapsed.

She, like many, had always thought that mathematics did not derive its meaning from the universe, but rather imposed some meaning onto the universe. Physical entities were not greater or less than one another, not similar or dissimilar; they simply were, they existed. Mathematics was totally independent, but it virtually provided a semantic meaning for those entities, supplying categories and relationships. It didn't describe any intrinsic quality, merely a possible interpretation.

But no more. Mathematics was inconsistent once it was removed from physical entities, and a formal theory was nothing if not consistent. Math was *empirical*, no more than that, and it held no interest for her.

What would she turn to, now? Renee had known someone who gave up academia to sell handmade leather goods. She would have to take some time, regain her bearings. And that was just what Carl had been trying to help her do, throughout it all.

8b

Among Carl's friends were a pair of women who were each other's best friend, Marlene and Anne. Years ago, when Marlene had considered suicide, she hadn't turned to Anne for support: she had turned to Carl. He and Marlene had sat up all night on a few occasions, talking or sharing silence. Carl knew that Anne had always harbored a bit of envy for what he had shared with Marlene, that she had always wondered what advantage he held that allowed him to get so close to her. The answer was simple. It was the difference between sympathy and empathy.

Carl had offered comfort in similar situations more than once in his lifetime. He had been glad he could help, certainly, but more than that, it had felt right to sit in the other seat, and play the other part.

He had always had reason to consider compassion a basic part of his character, until now. He had valued that, felt that he was nothing if not empathic. But now he'd run up against something he'd never encountered before, and it rendered all his usual instincts null and void.

If someone had told him on Renee's birthday that he would feel this way in two months' time, he would have dismissed the idea instantly. Certainly such a thing could happen over years; Carl knew what time could do. But two months?

After six years of marriage, he had fallen out of love with her. Carl detested himself for the thought, but the fact was that she had changed, and now he neither understood her nor knew how to feel for her. Renee's intellectual and emotional lives were inextricably linked, so that the latter had moved beyond his reach.

His reflex reaction of forgiveness cut in, reasoning that you couldn't ask a person to remain supportive through any crisis. If a man's wife were suddenly afflicted with mental illness, it would be a sin for him to leave her, but a forgivable one. To stay would mean accepting a different kind of relationship, something which not everyone was cut out for, and Carl never condemned a person in such a situation. But there was always the unspoken question: What would I do? And his answer had always been, I would stay.

Hypocrite.

Worst of all, he had been there. He had been absorbed in his own pain, he had tried the endurance of others, and someone had nursed him through it all. His leaving Renee was inevitable, but it would be a sin

he couldn't forgive.

9

Albert Einstein once said, "Insofar as the propositions of mathematics give an account of reality they are not certain; and insofar as they are certain they do not describe reality."

9a = 9b

Carl was in the kitchen, stringing snow pea pods for dinner, when Renee came in. "Can I talk to you for a minute?"

"Sure." They sat down at the table. She looked studiedly out the window: her habit when beginning a serious conversation. He suddenly dreaded what she was about to say. He hadn't planned to tell her that he was leaving until she'd fully recovered, after a couple of months. Now was too soon.

"I know it hasn't been obvious--"

No, he prayed, don't say it. Please don't.

"--but I'm really grateful to have you here with me."

Pierced, Carl closed his eyes, but thankfully Renee was still looking out the window. It was going to be so, so difficult.

She was still talking. "The things that have been going on in my head---" She paused. "It was like nothing I'd ever imagined. If it had been any normal kind of depression, I know you would have understood, and we could have handled it."

Carl nodded.

"But what happened, it was almost as if I were a theologian proving that there was no God. Not just fearing it, but knowing it for a fact. Does that sound absurd?"

"No."

"It's a feeling I can't convey to you. It was something that I believed deeply, implicitly, and it's not true, and I'm the one who demonstrated it."

He opened his mouth to say that he knew exactly what she meant, that he had felt the same things as she. But he stopped himself: for this was an empathy that separated rather than united them, and he couldn't tell her that.

"Division by Zero" was first published in Full Spectrum 3 (Bantam Spectra, 1991). It can be found, along with seven other great stories, in Ted Chiang's recent collection Stories of Your Life and Others (Tor, 2002).

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